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Modelling and parameter estimation for fly-by-wire aircraft/control systems

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Abstract. This paper covers in detail the issues related to parameter estimation of open loop dynamics of fly-by-wire aircraft/control systems from closed loop data. System identifiability aspects in the closed loop and the effect of various feedback types on the parameterisation of the system matrices are reviewed. The methods commonly employed for the detection of collinearity in the data are discussed. A brief discussion of the common methods used for analysis of unstable/augmented aircraft are given. Also, controller information based identification method (CIBIM), which utilises knowledge of the controller in the analysis, is presented. The discussion is followed by numerical results of application of the techniques to simulated data.

Keywords. Fly-by-wire aircraft; control systems; open loop dynamics; parameter estimation; filtering; regression.

1. Introduction

Modelling and identification play very significant roles in present day analysis of complex dynamical systems. Mathematical modelling of any system is necessary to understand its static/dynamical behaviour. The structure of the mathematical model involves parameters characterising the system and these are estimated using estimation techniques. Mathematical modelling of aerospace vehicles is very important since many applications require such information in the form of aerodynamic derivatives that appear in the mathematical model, and which are required for the following reasons (Maine & Iliff 1986).

(i) For explaining aerodynamic stability and control behaviour of the vehicle, thereby describing its static/dynamic behaviour;
(ii) for design of flight control systems, and
(iii) for high fidelity simulators.

Earlier attempts on parameter estimation of linear/nonlinear aerospace vehicles are well reported in the open literature (Jategaonkar & Plaetschke 1983). For stable aircraft applications, most of these methods (Girija & Jategaonkar 1991) work well. Some methods are only suitable for identification of transfer functions of a plant operating in a closed loop control system. In many situations, estimation of parameters of the state space models is required. Determination of aerodynamic derivatives of an aircraft with a fly-by-wire control system (FBWCS, unstable aircraft augmented by feedback) poses new challenges to aircraft identification and parameter estimation problem. When identification experiments are performed with an aircraft operating in a closed loop, the feedback introduces correlations among the input and output variables. This data correlation might cause identifiability problems that render estimates of some derivatives inaccurate (Belsely *et al* 1980; Klein 1989). Due to feedback action, constantly trying to generate controlled responses, the measured responses may not display the modes of the vehicle.

For many applications in the field of flight mechanics, the prediction and determination of the parameters of mathematical models of unstable aircraft is a primary requirement. However, the determination of parameters of open loop plant from the closed loop data presents a variety of problems even when the basic plant is stable. The problem complexity increases when the basic plant is unstable due to the fact that the integration of the state model leads to numerical divergence. The problem is further compounded, if data are noisy, which is the usual case. There are mainly three approaches to the problem of parameter estimation of unstable/augmented systems.

(1) Open loop data can be used directly, ignoring the effect of feedback and, if the feedback loop is tight, this method may give estimates with large uncertainty due to data collinearity (Klein 1989).

(2) An 'optimal input signal' could be designed/generated taking into account the presence of feedback and then the closed loop data could be analysed (Mereau & Abu El Ata-Doss 1985).

(3) Input/output data of the closed loop system with the complete model of the entire system can be used. This involves modelling of control system blocks and nonlinearities. This approach is complicated as it involves higher order system models to be used in the estimation procedure (Koehler & Wilhelm 1979).

In this paper, system identifiability issues in closed loop and the effect of various feedback types on the parameterisation of the system matrices are reviewed. The methods commonly employed for the detection of collinearity in the data are discussed. A brief discussion of the common methods, namely (i) filter error method (FEM), (ii) *ad hoc* stabilisation in output error method, (iii) mixed estimation, and (iv) principal component regression methods, used for analysis of unstable/augmented aircraft is presented. Numerical results of mixed estimation (Girija & Raol 1995a) applied to simulated data and unit uppertriangular diagonal (UD) factorisation-based Kalman filtering applied to unstable systems (Girija & Raol 1993) are presented.

Also, in situations where the control system details are known, they can be utilised in the aggregate model of the system leading to the controller information based identification method (CIBIM) (Girija & Raol 1995b). One of the variants of this method is to estimate equivalent derivatives of the entire system and use the knowledge of the controller to retrieve the open loop dynamics. The results of this method for an unstable aircraft with a simple feedback loop are presented in this paper.

2. Identifiability issues for closed loop systems

The general block diagram of a system operating in closed loop configuration is shown in figure 1. The input (at J), the error signal input (at K) to the plant and the output (at L) are generally measured. The parameters of the plant are to be estimated from these measurements. Two procedures are possible by which the parameters can be estimated from the measured data: (1) Direct identification – a chosen identification method is applied to the data between points K and L, ignoring the presence of the feedback; (2) Indirect identification – the data between points J and L can be used with a chosen identification method to generate equivalent derivatives. Here the closed loop system is regarded as a whole and the parameters are estimated. Using the knowledge of the feedback gain/controllers, the parameters of the plant can be retrieved from the equivalent model. Alternatively, the known models of control systems can be used along with the plant model and unknown parameters of the plant can be determined.

When the direct identification method is used to estimate the parameters of the aircraft operating in closed loop, the feedback introduces correlations among the input and output variables. Noise correlations are also present. This data/noise correlation causes identifiability problems which render estimates of some derivatives inaccurate (Koehler & Wilhelm 1979; Klein 1989). The correlations between the aircraft states and control inputs lead to high correlations between the corresponding stability and control derivatives such that not all the derivatives can be estimated independently (Maine & Murray 1988). Hence, only a degenerate model is identifiable by fixing some of the derivatives at their predicted values, which might result in incorrect estimates in case of wrong predictions.

The control system blocks, namely the feedforward and feedback filters, augment the unstable plant. The effect of feedback on the parameters and structure of the mathematical model is shown in table 1 with the basic plant description given as (Koehler & Wilhelm 1979):

$$\dot{x} = Ax + Bu, \tag{1}$$

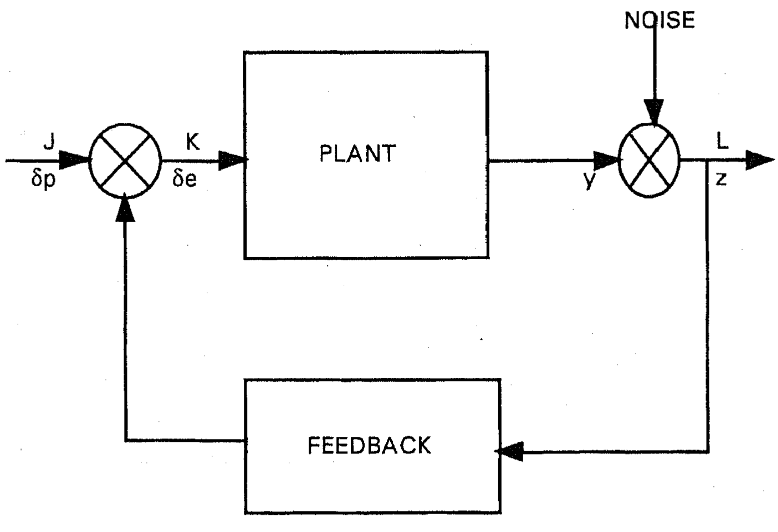


Figure 1. General block diagram of augmented system.

Table 1. Effect of feedback on the parameters and structure of the math model.

Control system type	Input description	Modified sytem	Remarks (changes in)
Constant feedback	$u = Cx + Du_s$	$\dot{x} = (A + BC)x + BDu_s$	Coefficients in the column of feedback
Differential feedback	$u = Cx + E\dot{x} + Du_s$	$\dot{x} = (1 + BE)^{-1} \{(A + BC)x + BDu_s\}$	Almost all coefficients
Integrating feedback	$\dot{u} + Fu = Cx + Du_s$	$\begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ C & F \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ D \end{bmatrix} u_s$	Structure itself

where x is the $(n \times 1)$ state vector. In table 1, u_s represents the input at point J (figure 1), A is the system matrix, B the control matrix, C the feedback matrix, D the feedforward matrix, and E and F the matrices associated with differential feedback and integrating feedback respectively. From table 1 it is clear that the control system with constant feedback does not affect the structure of the system, but affects only the estimates of the elements of the system matrix A. The modified equations represent a system having a different state matrix. With differential feedback, although the basic structure is the same, all the coefficients are affected even if only one signal is fedback. When the feedback control system has integrators in the feedback loops, the entire structure is changed and the number of poles increases with the number of equations and for very highly augmented systems the order can be very high.

Equation (1) is modified to include the noise:

$$\dot{x} = Ax + Bu + w. \tag{2}$$

If we assume that the feedback is of proportional type (or if the control system dynamics are only weakly excited during the measurement period), the signal u can be represented by

$$u = Cx + Du_s. \tag{3}$$

It is required to estimate the parameters of A, B, C and D. Multiplying (3) by an arbitrary matrix Λ and adding to (2), we get

$$\dot{x} = (A + \Lambda C)x + (B - \Lambda)u + w + \Lambda Du_s. \tag{4}$$

The parameter estimation method treats $(w + \Lambda Du_s)$ as noise and determines the coefficients by minimising the effect of this noise. If the input u_s large, the elements of Λ become insignificant and hence may be neglected. In such a case, (2) and (4) are identical and consequently the feedback has very little influence on the estimated results. However, the large u_s might take the system into nonlinear regions of its behaviour. If the input u_s is small or of short duration, Λ influences the coefficients matrices of x and u. The results of identification will be $(A + \Lambda C)$ and $(B - \Lambda)$ instead of A and B.

When the aircraft responses are correlated due to augmentation system, we have

$$x = Gx, \quad G \neq I, \tag{5}$$

with G as the matrix whose elements could be the feedback gains. Inserting (5) into (2) we get,

$$\dot{x} = [A + \Lambda(G - I)]x + Bu + w. \tag{6}$$

Even here it is difficult to determine the elements of A from output responses, due to the fact that Λ is an arbitrary matrix.

Thus it is clear that any control augmentation causes 'near linear' relationships among variables which are used in the estimation algorithm. It is this collinearity which could affect the accuracy of the estimates. Hence it is required to detect this collinearity in the data and use an appropriate estimation procedure.

3. Collinearity and its detection

An aircraft aerodynamic model can be written in general form as (Klein 1989),

$$y = \theta_0 + \theta_1 x_1 + \cdots + \theta_n x_n, \tag{7}$$

where x_j, $j = 1, 2, \ldots, n$ are the regressors (state and input variables or their combinations), y is a dependent variable (the aerodynamic force or moment coefficient) and $\theta_0 \cdots \theta_n$ are the unknown aerodynamic coefficients. When measured y and x are used, the regression equation takes the form,

$$Y = X\theta + \varepsilon, \tag{8}$$

where Y is $(N \times 1)$ measurement vector, X is the $(N, n+1)$ matrix of regressors and ones, (the ones to account for the constant term in any regression equation), and θ, the $(n+1, 1)$ unknown parameter vector.

The least squares estimates of the parameters θ can be obtained as:

$$\hat{\theta}_{LS} = (X'X)^{-1}X'Y. \tag{9}$$

Regressors X are generally handled by centreing and scaling them to unit length. If X_j^* denotes the columns of the normalised matrix, collinearity means that

$$\sum_{j=1}^{n} k_j X_j^* = 0, \tag{10}$$

for a set of constants k_j not all equal to zero. This collinearity causes computational problems because of ill-conditioning of the matrix and hence inaccurate estimates. For the detection of collinearity in the data X, the methods generally adopted are discussed next.

3.1 *Correlation matrix of regressors*

High correlation coefficient between two regressors can point to a possible collinearity problem. However, if there are several co-existing near-dependencies among regressors, the correlation matrix may be unable to detect collinearity (Belsely *et al* 1980).

3.2 *Eigensystem analysis and singular value decomposition*

For eigensystem analysis (Belsely *et al* 1980), the matrix $X'X$ is decomposed as

$$X'X = M\Delta M'. \tag{11}$$

Here, $'$ denotes transpose of the matrix/vector, Δ is $(n \times n)$ diagonal matrix with its elements as the eigenvalues λ_j of $X'X$ and M is an $(n \times n)$ orthogonal matrix with its columns as the eigenvectors of $X'X$. Eigenvalues close to zero indicate nearly linear dependency in the data. Since it is difficult to specify how small the eigenvalue should be, condition number, which is defined as,

$$K_j = |\lambda_{\max}|/|\lambda_j|, \tag{12}$$

is used as a measure of collinearity. Condition number exceeding 1000 is an indication of severe collinearity.

Because of its better numerical properties, singular value decomposition of matrix X is used to detect collinearity. Here the matrix X is decomposed as

$$X = ZSV', \tag{13}$$

where Z is an $(N \times n)$ matrix and $Z'Z = V'V = I$.

Here, S is an $(n \times n)$ diagonal semi-positive definite matrix with its elements as the singular values of X. Condition index which is defined as

$$\eta_j = \mu_{\max}/\mu_j, \tag{14}$$

is used as a measure of collinearity; values between 5 and 10 indicate mild collinearity and those between 30 and 100 indicate strong collinearity.

3.3 *Parameter variance decomposition*

In this method, the variance of each parameter is decomposed into a sum of components, each corresponding to one and only one of the singular values. The covariance matrix of the parameter estimates θ is given by

$$\mathrm{Cov}(\hat{\theta}) = \sigma_r^2 (X'X)^{-1} = \sigma_r^2 M \lambda^{-1} M', \tag{15}$$

where σ_r^2 is the residual variance.

The variance of each parameter is equal to

$$\sigma_{\theta_j}^2 = \sigma_r^2 \sum_{k=1}^{n} \frac{t_{jk}^2}{\lambda_j} = \sigma_r^2 \sum_{k=1}^{n} \frac{t_{jk}^2}{\mu_j^2}, \tag{16}$$

where t_{jk} are the elements of the eigenvector t_j associated with λ_j Equation (16) decomposes the variance of each parameter into a sum of components, each corresponding to one and only one of the n singular values. Since μ_j appears in the denominator, one or more small singular values can increase the variance of θ_j. Hence an unusually high property of the variance of two or more coefficients for the same small singular value can provide evidence that the corresponding near-dependency causes problems.

Defining

$$\phi_{jk} = t_{jk}^2/\mu_j^2 \quad \text{and} \quad \phi_j = \sum_{k=1}^{n} \phi_{jk}, \tag{17}$$

one gets the j, k variance-decomposition proportion as the proportion of the variance of the jth regression coefficient associated with the kth component of its decomposition in (17) and is given by

$$\Pi_{kj} = \phi_{jk}/\phi_j; \quad j, k = 1, 2, \ldots, n. \tag{18}$$

Since two or more regressors are required to create near-dependency, two or more variances will be adversely affected by high variance-decomposition proportions associated with a singular value. Variance proportions greater than 0.5 are recommended as guidelines for possible collinearity problems (Belsely *et al* 1980).

4. Methods for parameter estimation of unstable/augmented systems

The output error method (OEM) is most widely used for the determination of stability and control derivatives of aircraft. OEM requires numerical integration of unstable equations of motion of the plant and this in turn causes numerical divergence. Hence, although in principle, OEM is applicable to unstable system analysis, special care has to be taken to avoid the problem of numerical divergence of the algorithm. One such method it to employ *ad hoc* stabilisation in OEM (Maine & Murray 1988). An alternative is to use the filter error method (Jategaonkar & Plaetschke 1989). OEM is a special case of the filter error method described below.

4.1 *Filter error method*

The filter error method (FEM) is practical for linear systems which have the general form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w; \quad x(0) = x_0; \quad w \sim N(0, FF'), \\ z(t_i) &= Hx(t) + v, \quad i = 1, 2, \ldots, N; \quad v = N(0, GG'). \end{aligned} \tag{19}$$

The maximum likelihood estimates of $\theta = \{A, B, H, G\}$ for this system are obtained by minimising the cost function:

$$J(\Theta) = \sum_{i=1}^{N} [z(t_i) - \hat{z}(t)]^T R^{-1} [z(t_i) - \hat{z}(t_i)], \tag{20}$$

where R is the innovation covariance matrix and $\hat{z}(ti)$ is predicted using the (steady state) Kalman filter (Jategaonkar & Plaetschke 1989):

$$\hat{x}(t_0) = \hat{x}_0, \tag{21}$$

$$\tilde{x}(t_i) = \phi\tilde{x}(t_{i-1}) + B_d u(t_{i-1}), \tag{22}$$

$$\tilde{z}(t_i) = H\tilde{x}(t_i), \tag{23}$$

$$\tilde{x}(t_i) = \tilde{x}(t_i) + K[z(t_i + K[z(t_i) - \tilde{z}(t_i)], \tag{24}$$

where θ is the transition matrix and B_d the discrete control matrix, '$\sim$' represents the predicted variables and '$\wedge$' the estimated variables.

The steady state Kalman gain is given by,

$$K = PH'(HPH' + GG')^{-1}, \tag{25}$$

where P is the steady-state covariance of the predicted state, given as the limit of the Ricatti equation (Maine & Murray 1989):

$$P_{i+1} = \phi[P_i - P_i H'(HP_i H' + GG')^{-1} HP_i]\phi' + FF'. \tag{26}$$

The above equation converges to a unique solution independent of the initial covariance and the filter is stable if all the unstable modes of the system are observable and controllable. Filter error method reduces to OEM for stable systems with no process noise. For unstable systems with $FF' = 0$, the solution is not unique. For nonsingular FF', steadystate solutions are unique and smoothly approach the nonzero steady state solution. OEM works with $F = 0$, if all the aircraft modes are observable and there are no neutrally stable modes. The limitation of FEM is that it is only practical for linear systems and involves considerable programming effort.

4.2 *Ad hoc stabilisation method in OEM*

The stabilisation of the filter in FEM is due to the feedback term which is proportional to the error between the measured and the estimated states (also called fit error). The filter feedback helps stabilise the analysis algorithm and does not bear any relation to the control system feedbacks that stabilise the aircraft. Stabilising feedback of fit errors can alternatively be implemented in the mathematical models used in identification software. These *ad hoc* feedback terms could include nonlinearities and other practical engineering judgement to improve the results.

The *ad hoc* method has a form somewhat similar to FEM and gives similar results but is applicable to many practical situations if proper care is taken to obtain appropriate feedback gains. For the linear system described by (13), the predicted state is:

$$\tilde{x}(t_i) = \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}), \tag{27}$$

$$\tilde{z}(t_i) = H\tilde{x}(t_i). \tag{28}$$

By introducing *ad hoc* stabilisation by software feedback of a state in the mathematical model, we have the input to the model:

$$u(t_{i-1}) = u(t_{i-1}) + K_{SF}\tilde{x}(t_{i-1}), \tag{29}$$

where K_{SF} is the software gain. Hence,

$$\begin{aligned}\tilde{x}(t_i) &= \phi\tilde{x}(t_{i-1}) + B_d u(t_{i-1}) + B_d K_{SF}\tilde{x}(t_{i-1})\\ &= (\phi + B_d K_{SF})\tilde{x}(t_{i-1}) + B_d u(t_{i-1}).\end{aligned} \tag{30}$$

If K_{SF} is properly chosen, the system $(\phi + B_d K_{SF})$ is made locally stable in the mathematical model of the original plant ϕ which may be unstable.

In FEM, substituting (22) in (24), we have,

$$\hat{x}(t_i) = \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}) + KH[x(t_i) - \tilde{x}(t_i)]. \tag{31}$$

Simplifying after substituting for $\tilde{x}(t_i)$ we have (ignoring the noise terms):

$$\begin{aligned}\hat{x}(t_i) &= \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}) + KH\phi[x(t_{i-1}) - \hat{x}(t_{i-1})]\\ &= (I - KH)\phi\hat{x}(t_{i-1}) + KH\phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}).\end{aligned} \tag{32}$$

Equations (30) and (32) have somewhat similar form. Hence it follows that in the adhoc stabilisation method, (30), the software feedback produces a stabilising effect similar to that produced by the feedback of the fit error in the Kalman filter in FEM.

4.3 *Mixed estimation*

The method of mixed estimation (Klein 1989) augments the measured data by *a priori* information on the parameters directly. Assuming that $p \leq n$ (n is the total number of parameters to be estimated) prior information on the elements of θ are available, the *a priori* information equation (PIE) can be formulated as,

$$a = C\theta + \zeta, \tag{33}$$

where a is a p-vector of known *a priori* values, C is a matrix of each $p \leq n$ which includes known constants and ζ is a random vector with $E(\zeta) = 0$, $E(\zeta\varepsilon') = 0$, $E\{\zeta\zeta\} = \sigma^2 W$, where W is a known weighting matrix. Here E stands for mathematical expectation.

Combining (8) and (33), the mixed model is given by

$$\begin{bmatrix} Y \\ a \end{bmatrix} = \begin{bmatrix} X \\ C \end{bmatrix}\theta + \begin{bmatrix} \varepsilon \\ \zeta \end{bmatrix}. \tag{34}$$

Applying the least squares method to (34), the mixed estimates (ME) are obtained as

$$\hat{\theta}_{\mathrm{ME}} = (X'X + C'W^{-1}C)^{-1}(X'Y + C'W^{-1}a), \tag{35}$$

with the covariance matrix given by

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{ME}}) = \sigma^2[X'X + C'W^{-1}C]^{-1}. \tag{36}$$

In practical applications, PIE may not be known exactly and hence the resulting estimator is biased. Generally the W matrix is diagonal with the elements representing uncertainty of *a priori* values.

4.4 *Principal components regression method*

Here the original regressors x_j are transformed into the space of orthogonal regressors z_j (Klein 1989). The regression model after transformation becomes,

$$Y = Z\gamma + \varepsilon, \tag{37}$$

where $Z = XT$ and $\theta = T\gamma$, T is the eigenvector matrix of the $X'X$ matrix and γ is the set of parameters associated with the orthogonal regressors. Columns of Z matrix are called 'Principal Components'. The estimates are obtained by

$$\hat{\gamma} = (Z'Z)^{-1}Z'Y = \Lambda^{-1}Z'Y, \tag{38}$$

and the covariance matrix of $\hat{\gamma}$ is

$$\mathrm{Cov}(\hat{\gamma}) = \sigma^2\Lambda^{-1}. \tag{39}$$

To get the principal component estimates, the regressors are arranged in order of decreasing eigenvalues λ_j, the principal components associated with small eigenvalues are deleted and the LS is applied to the remaining components. The principal component estimator has the form

$$\hat{\theta}_{PC} = \sum_{j=1}^{n-r} (1/\lambda_j)' t_j' X' Y t_j. \tag{40}$$

When all data are ill conditioned, the principal component regression yields better estimates than the conventional LS techniques.

In this paper, the mixed estimation technique is used for parameter estimation along with the collinearity diagnostics. This is quite suitable for aircraft applications, since the *a priori* information required for this method is available from wind tunnel/analytical predictions or from previous flight test experiments/analysis.

4.5 *Numerical validation of ME method*

All the collinearity diagnostics described in § 3 are implemented in PC MATLAB. The least squares mixed estimation (LSME) method is also implemented in PC MATLAB. OEM program, based on the maximum likelihood method (Jategaonkar & Plaetschke 1983), is used to analyse the simulated data to generate results for comparison purposes for augmented dynamical systems.

Figure 2 shows the block diagram of a typical augmented longitudinal dynamics of an aircraft. Block 1 in the figure is for pilot command shaping, block 2 is actuator dynamics with delay, block 3 represents the aircraft dynamics and blocks 4–7 are feedback filters.

Example 1. Second order short period dynamics of an aircraft having the following state and observation equations are considered for generating simulated data.

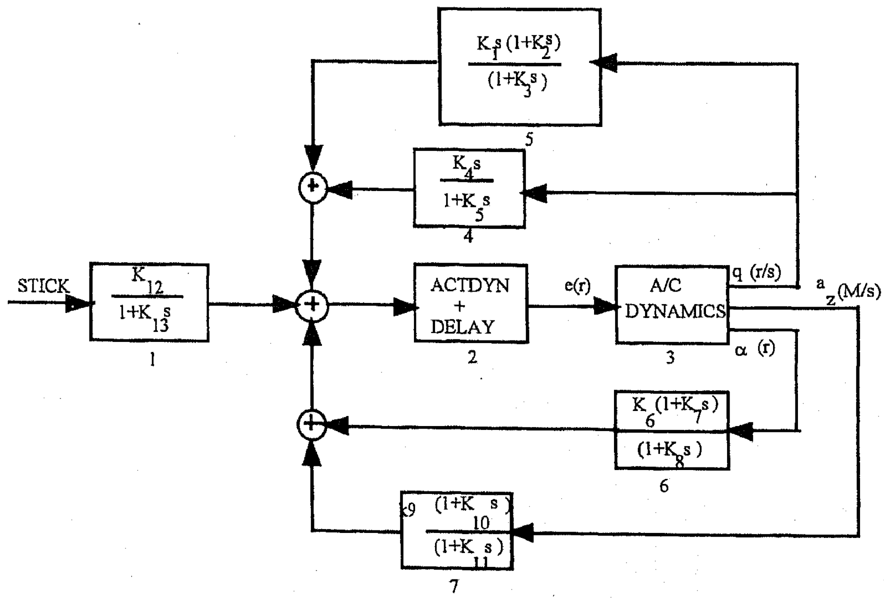


Figure 2. Block diagram of simulated closed loop system.

State equations:

$$\dot{\alpha} = Z_\alpha \alpha + q + Z_{\delta_e} \delta_e, \tag{41}$$
$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e} \delta_e. \tag{42}$$

Observation equations:

$$\begin{aligned} \alpha_m &= \alpha, \\ q_m &= q, \end{aligned} \tag{43}$$

where α is the angle of attack, q is the pitch rate and δ_e is the elevator (control surface) deflection. Z_α, Z_{δ_e}, M_α, M_q, M_{δ_e} are the aerodynamic derivatives to be estimated. For generating simulated data, only the α signal is fed back to the control surface through a gain K_α so that we have the feedback signal given by

$$\delta_e = K_\alpha \alpha + \delta_p. \tag{44}$$

By varying the gain K_α different sets of data are generated for analysis. Typical time histories of α, q, δ_e, when the feedback gain $=1$, and δ_p (pilot input) is a doublet signal, are shown in figure 3 .Sampling period $= 0.1$ s and the data record length of 10 s is used in the analysis.

The closed loop eigenvalues of the system are shown in table 2 for a range of the feedback gain ($K_\alpha = 0$ to 1). The parameters estimated using OEM and LS from the open loop ($K_\alpha = 0$) and SNR$=\infty$ and SNR$=10$ are given in table 3. The parameter estimation error norms (PEEN) are defined as,

$$\mathrm{PEEN}(\%) = 100^*\mathrm{norm}(b_t - b_e, i)/\mathrm{norm}(b_t, i), \tag{45}$$

where b_t = vector of true parameters, b_e = vector of estimated parameters, and if $i = 1$, it is L1 norm and for I $= 2$ it is L2 norm.

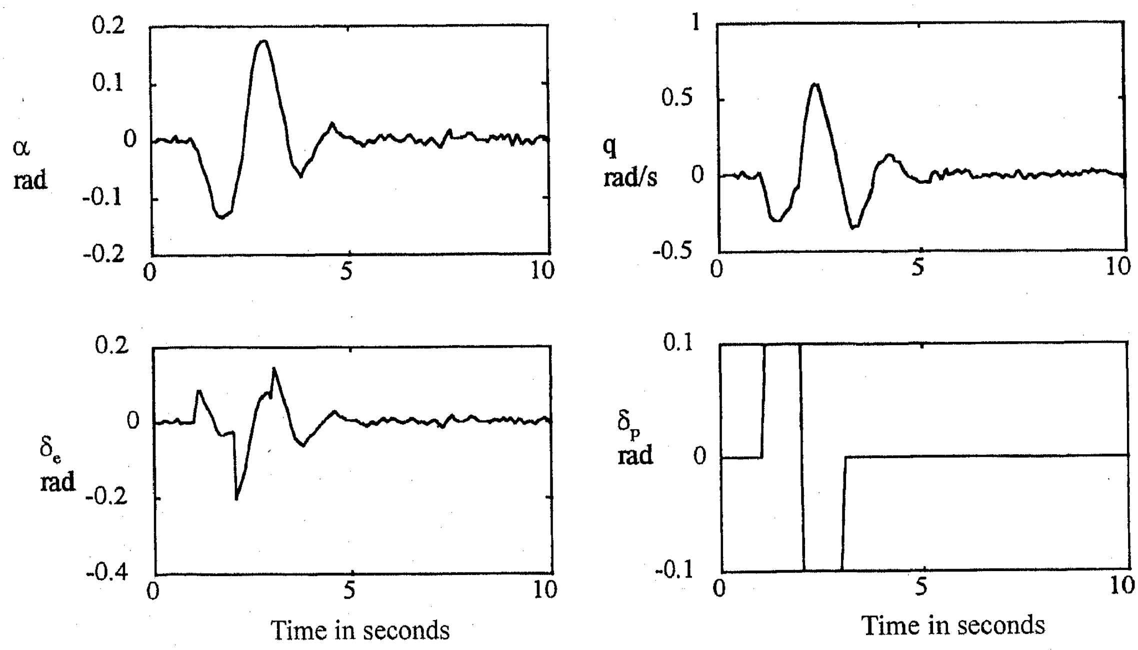


Figure 3. Time history plots (example 1, second-order short period dynamics).

Table 2. Closed loop eigenvalues of the second order system (example 1).

Gain K_α	Eigenvalues	ω_n rad/s	ζ
0.0	$-0.2881, -1.7443$	–	–
0.03	$-0.7188, -1.3265$	–	–
0.07	$-1.0318 \pm j0.7256$	1.2164	0.8180
0.15	$-1.0485 \pm j1.2882$	1.6610	0.6312
0.3	$-1.0809 \pm j1.9609$	2.2390	0.4827
1.0	$-1.2315 \pm j3.7435$	3.9409	0.3125

The estimated derivatives using OEM and LS for the case when SNR $= \infty$ are close to the true values. As the SNR decreases, the OEM estimates show some deviations and the standard deviation of the parameter estimates increases. However, the LS estimates show a much larger deviation from the true values. This is because the LS method generates biased estimates when the regressors are noisy. The L1 and L2 norms also indicate an increase in error as the signal-to-noise ratio decreases.

Table 4 gives the estimates using OEM and LS (for $K_\alpha = 1$). For SNR$= \infty$, OEM estimates are close to true values whereas the LS estimates show larger deviations. As the SNR decreases, standard deviations of the parameters increase for OEM and LS methods, and the parameter estimates deviate from the true values. The L1 and L2 norms also increase as the SNR decreases. This behaviour can be attributed to the correlations of signals (and noise) due to feedback.

Since the data are generated using closed loop plant, the collinearity diagnostics described in § 3 are computed to assess the extent of collinearity in the data. The correlation matrix and variance proportions for one typical case of SNR$= 100$ are given in tables 5 and 6. The correlation matrix and variance proportions are computed assuming there is a constant term in the regression equation in addition to the states α, q, δ_e. In table 6, the condition numbers are also indicated. In figure 4 these values are plotted. The

Table 3. Parameter estimates (example 1, $K_\alpha = 0$).

		SNR			
		OEM		LS	
Parameter	True value	∞	10	∞	10
Z_α	-0.9624	-0.9602 (0.0004)	-0.8732 (0.07)	-0.9721 (0.037)	-0.4392 (0.45)
M_α	0.5273	0.5298 (0.0007)	0.5819 (0.11)	0.1947 (0.11)	-0.1200 (0.75)
M_q	-1.0698	-1.0743 (0.0005)	-1.2214 (0.10)	-0.8601 (0.07)	-1.7730 (0.49)
Z_{δ_e}	-0.4315	-0.4361 (0.0027)	-0.6084 (0.24)	-0.3844 (0.01)	-2.2503 (1.54)
M_{δ_e}	-14.5747	-14.5951 (0.003)	-15.5875 (0.73)	-13.0695 (0.32)	-9.5054 (2.56)
L1%	–	0.1947	8.4535	11.9800	55.8076
L2%	–	0.1477	7.1241	10.6200	38.80

Table 4. Parameter estimates (example 1, $K_\alpha = 0$).

		SNR			
		OEM		LS	
Parameter	True value	∞	10	∞	10
Z_α	−0.9624	−0.9449 (0.002)	−1.6962 (0.14)	−1.0072 (0.011)	−0.5913 (0.58)
M_α	0.5273	0.5517 (0.004)	0.1098 (2.38)	−0.9466 (0.31)	−3.4200 (1.65)
M_q	−1.0698	−1.1096 (0.0014)	−1.2476 (0.19)	−0.7360 (0.09)	−0.0400 (0.44)
Z_{δ_e}	−0.4315	−0.4535 (0.0035)	−1.2708 (0.11)	−0.3595 (0.01)	−0.7420 (0.71)
M_{δ_e}	−14.574	−14.784 (0.007)	−13.191 (1.09)	−12.310 (0.38)	−7.7420 (2.02)
L1%	–	1.7842	25.13	23.85	69.95
L2%	–	1.4778	16.077	18.58	53.19

Table 5. Correlation matrix (example 1, $K_\alpha = 1$).

Const. term	α	q	δe
1.0	−0.85	−0.36	−0.14
−0.85	1.0	0.67	−0.26
−0.36	0.67	1.0	−0.87
−0.14	−0.26	−0.87	1.0

correlation matrix indicates correlation of 0.67 between q and α It is 0.87 between q and δ_e. The variance proportions corresponding to the largest condition number also indicate collinearity between α, q, δ_e. Since the parameter most affected by feedback is M_α, it was decided to fix the corresponding control effectiveness parameter, M_{δ_e}, at a value equal to 0.5 times of its true value and use the LSME method for the same set of data. Table 7 gives the LSME estimates. These results indicate improvement in estimates for all cases. The results in table 7 when compared with those in table 3, indicate that the mixed estimation, where the *a priori* information is appended to the measurements, works efficiently when the SNR is high. The results of tables 3 and 7 are plotted in figure 5, where the parameter estimates for SNR$=\infty$ and SNR$=10$ are plotted. The LSME and OEM results for SNR$=\infty$ almost coincide for both $K_\alpha = 0$ and $K_\alpha = 1$, indicating that the effect of gain, when accurate measurements are available, is very small on the parameter estimates. The LS results deviate from the true values. This is to be expected of any regression (LS)

Table 6. Variance proportions (example 1, $K_\alpha = 1$).

Cond. No.	Const. term	α	q	δe
1	0.000	0.040	0.00	0.16
2.71	0.000	0.32	0.010	0.25
6.7	1.00	0.000	0.003	0.004
68.99	0.000	0.640	0.990	0.590

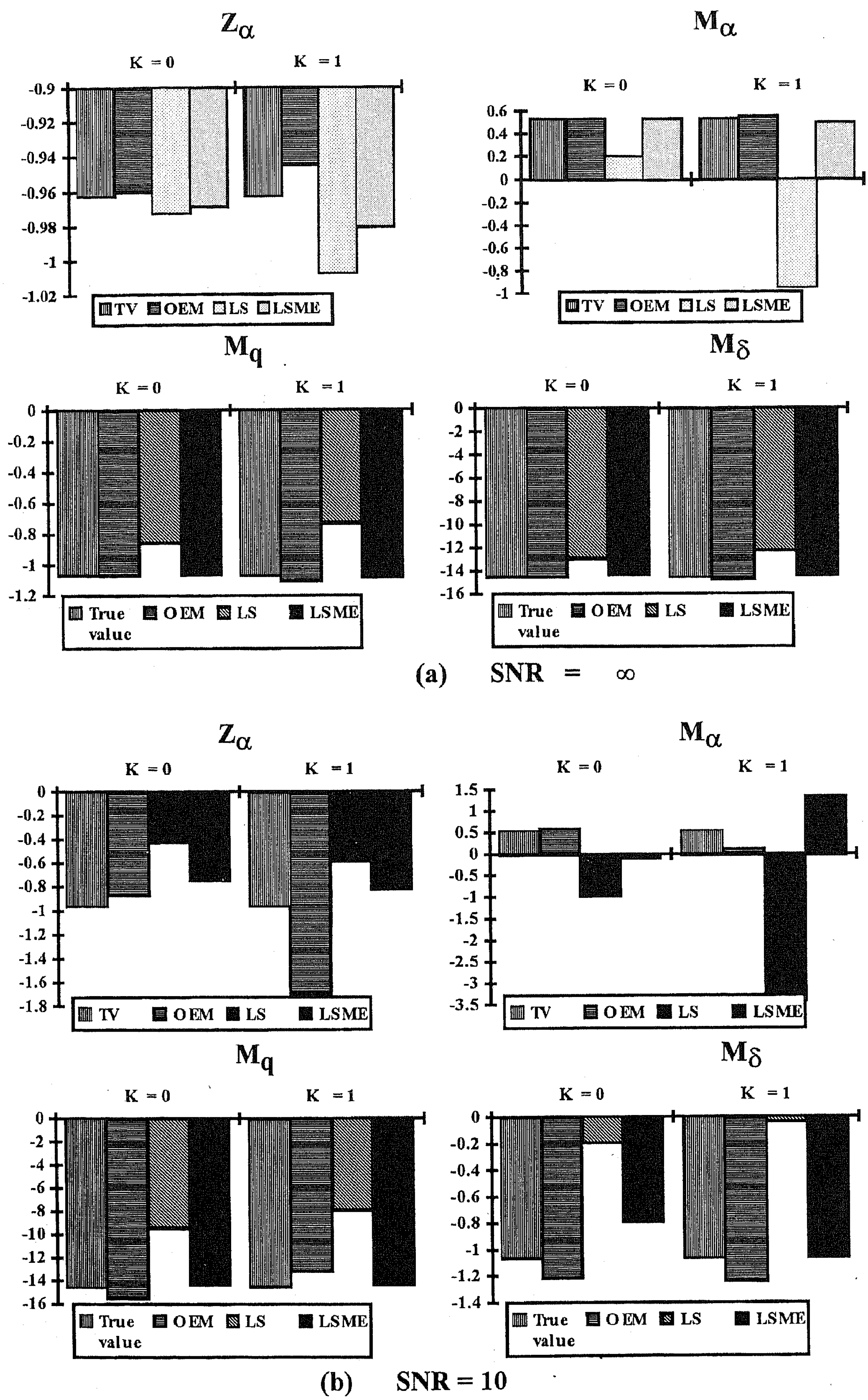
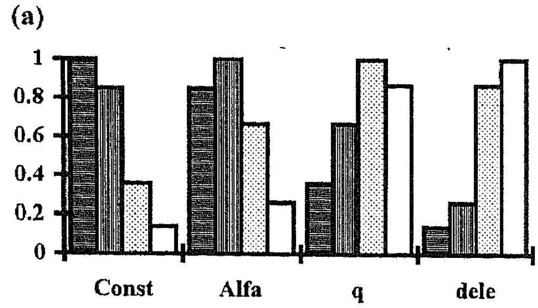


Figure 5. Parameter estimates compared with true values using OEM, LS and LSME methods (example).



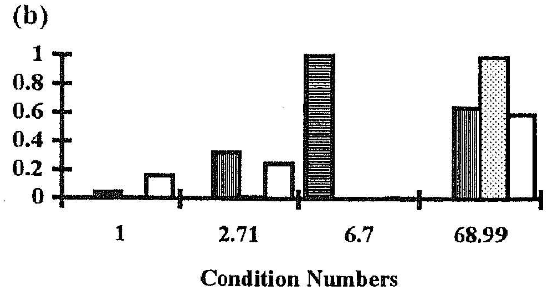


Figure 4. Correlation coefficients (a) and variance proportions (b) (example 1, second-order short period dynamics).

procedure where accurate measurements of state and control variables are necessary to get accurate estimates. Also, OEM results when SNR = 10, are closer to the true values for the no-feedback case but shows large deviation when $K_\alpha = 1$. However, LSME clearly gives closer estimates for all cases depicted in figure 5. In figure 6, PEENs are plotted.

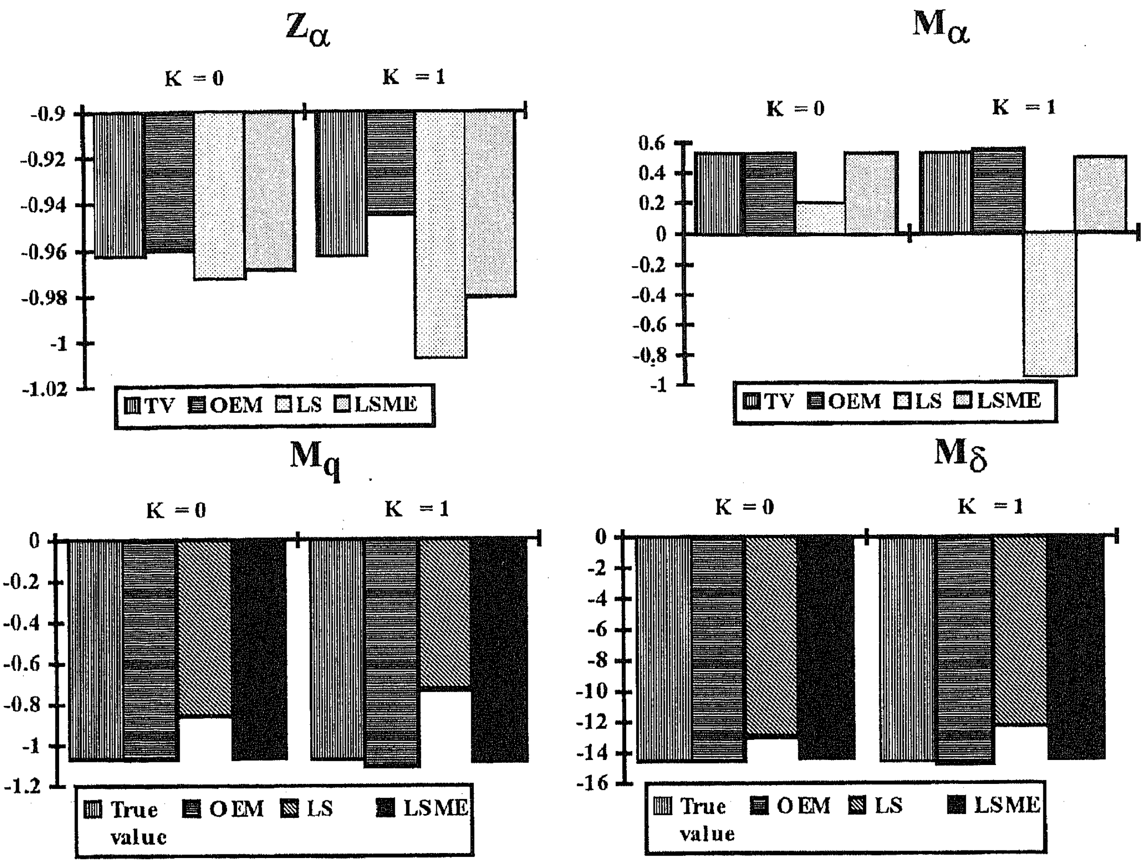
Example 2. The fourth-order longitudinal dynamics of an unstable aircraft and the associated filters in the feedback loops of figure 2 are simulated. A doublet pulse input is used to generate responses. The responses between the points K and L are used for estimating the aerodynamic derivatives of the aircraft. The state and observation equations are as below.

State equations:

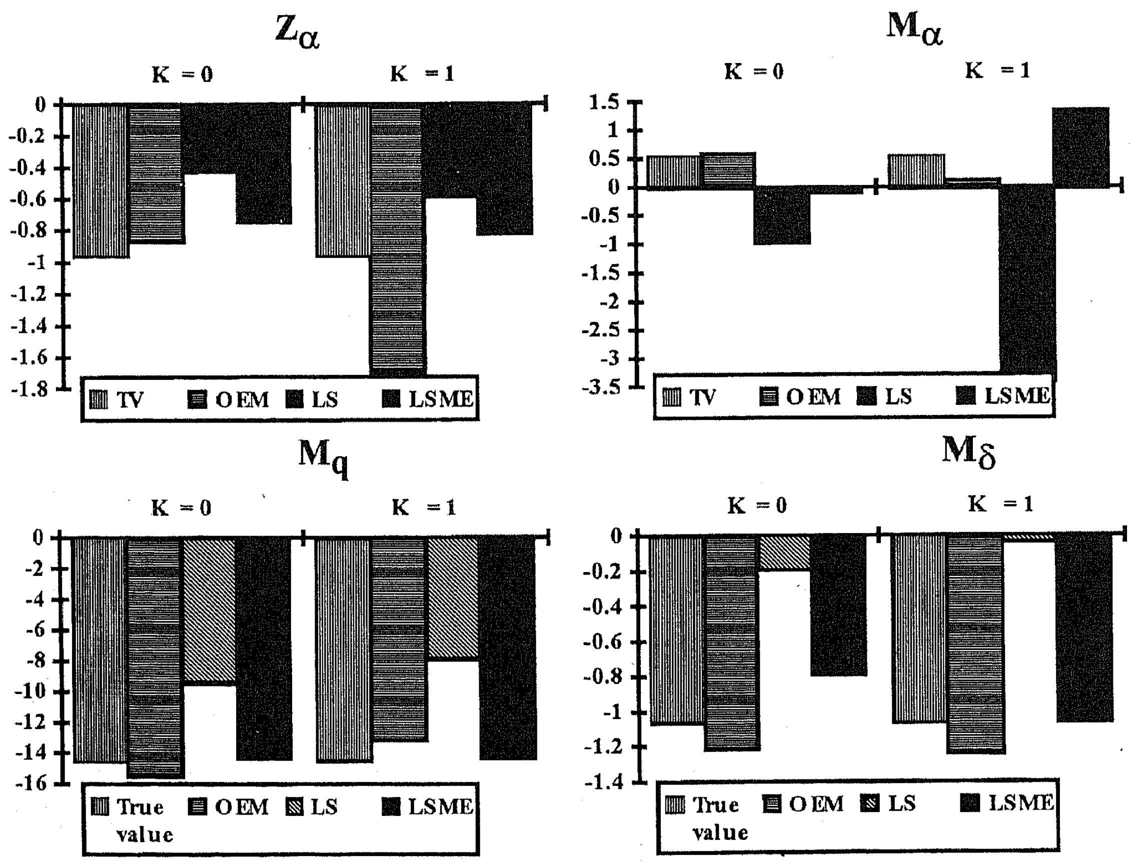
$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{v}/v_0 \end{bmatrix} = \begin{bmatrix} Z_{\alpha/v_0} & 1 & 0 & Z_{v/v_0} \\ M_\alpha & M_q & 0 & M_{v/v_0} \\ 0 & 1 & 0 & 0 \\ X_\alpha & 0 & X_\theta & X_{v/v_0} \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ v/v_0 \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \\ 0 \\ X_{\delta_e} \end{bmatrix} \delta_e. \tag{46}$$

Table 7. Parameter estimates using LSME method (example 1).

Parameter	$K_\alpha \rightarrow$ / SNR $\rightarrow$ / True value	0: ∞	0: 10	1: ∞	1: 10
Z_α	-0.9624	-0.9684 (0.003)	-0.7539 (0.350)	-0.9804 (0.011)	-0.8260 (0.34)
M_α	0.5273	0.5295 (0.09)	-0.1123 (0.54)	0.4936 (0.22)	1.3475 (0.98)
M_q	-1.0698	-1.0683 (0.05)	-0.7947 (0.05)	-1.0856 (0.07)	-1.0669 (0.33)
Z_{δ_e}	-0.4315	-0.4000 (0.0005)	-0.4052 (0.19)	-0.3999 (0.013)	-0.4006 (0.03)
M_{δ_e}	-14.5747	-14.4821 (0.04)	-14.4207 (0.28)	-14.4819 (0.04)	-14.4284 (0.19)
$L1\%$	–	3.5596	3.8519	3.5052	12.2892
$L2\%$	–	3.3220	2.4249	3.0868	11.7170

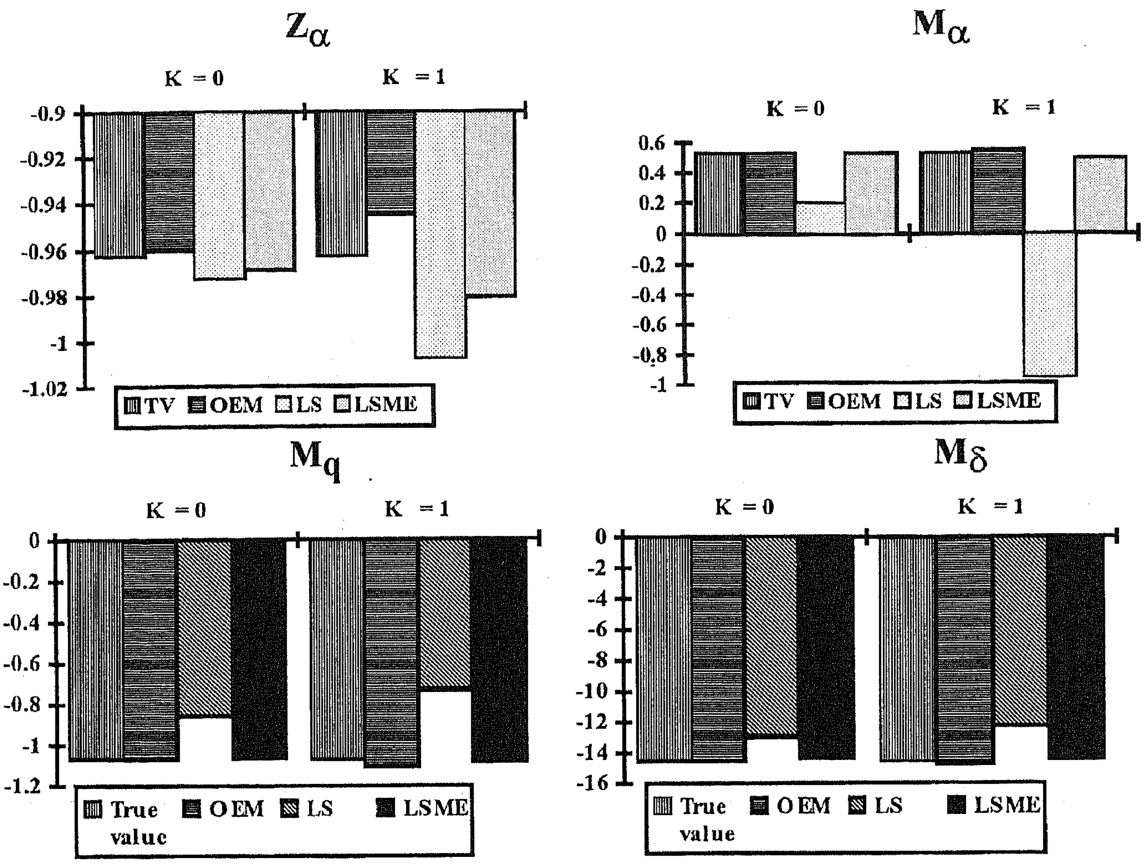


(a) SNR = ∞

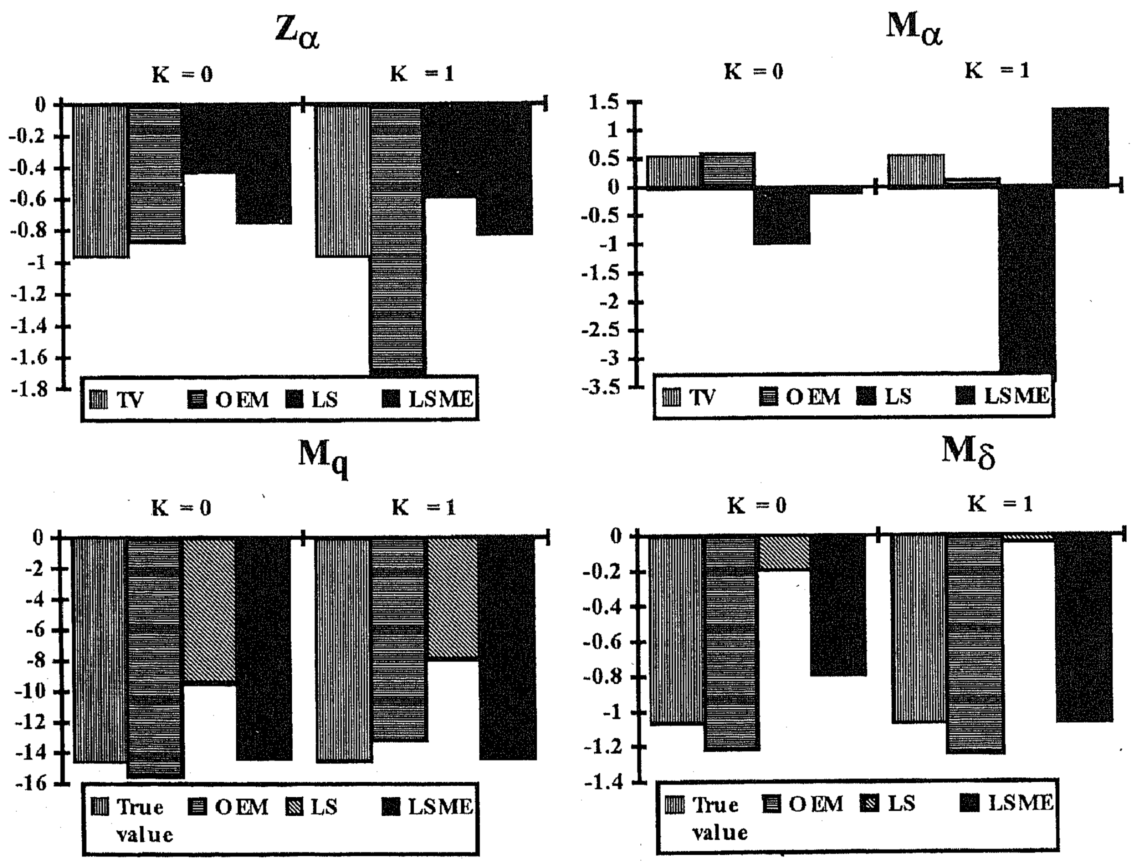


(b) SNR = 10

Figure 5. Parameter estimates compared with true values using OEM, LS and LSME methods (example).



(a) SNR = ∞



(b) SNR = 10

Figure 5. Parameter estimates compared with true values using OEM, LS and LSME methods (example).

Observation equations:

$$\begin{bmatrix} \alpha \\ q \\ a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ C_{31} & 0 & 0 & C_{34} \\ C_{41} & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ v/v_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ D_{31} \\ D_{41} \end{bmatrix} \delta_e, \tag{47}$$

where $Z_{(\cdot)}$, $X_{(\cdot)}$, $M_{(\cdot)}$, $C_{(\cdot)}$, $D_{(\cdot)}$ are the aerodynamic parameters to be estimated. (*Note*: The elements in the measurement matrices C and D are related to the aerodynamic derivatives in the state equations but have been estimated separately as additional parameters in the estimation procedure).

The eigenvalues of the open loop unstable plant are: -1.4314, $-1.617 \pm j(0.1037)$, 0.1151. There is one unstable pole on the right side of the s-plane. The eigenvalues of the closed loop system (10th order) are -7.8313 ± 8.2342, -9.9129, -3.2592, -0.7924, -0.0059 ± 0.0991, -0.4058 and -4.000. The control blocks are simulated with the gain values provided for nominal flight conditions. Time histories of simulated data (SNR = 100) are given in figure 7. The collinearity diagnostics for this data indicating the correlation matrix and variance proportions are given in tables 8 and 9.

The correlation matrix indicates correlation coefficients of 0.77 between the constant term and α, 0.996 between $v/v0$ and constant, 0.732 between $v/v0$ and q and 0.735 between δ_e and q. The variance proportions also indicate that the $v/v0$ term is correlated with the constant term as indicated by the value 0.87. The condition number of 1826 indicates the presence of high collinearity in this data. Table 10 shows the results of the LS and LSME with the $v/v0$ and the control effectiveness related derivatives kept fixed at 110% of the true value. The last column shows the estimates using OEM, which could not estimate all the derivatives, since it showed severe convergence problems. Figure 8 compares some of the estimates with the true values for SNR = 100 and SNR = 10. It is clear that for both the cases, the estimates using LSME are close to the true values. However, the M_α estimate (SNR = 10) shows considerable deviation even when LSME is used. This is to be expected of any LS procedure when noisy regressors are used for parameter estimation. The PEENs are seen to decrease when LSME method is used.

Thus it is clear that by using collinearity diagnostics and mixed estimation (*a priori* information for only those derivatives which introduce correlations), the method yields reasonably good and improved estimates even in cases where the feedback loops are complicated. Thus, it can be concluded that when correlation exists, its presence and effect can be ascertained and the use of ME to some extent enables one to get estimates in the presence of collinearity. However, in the presence of noisy measurements for states and control inputs, the LSME yields biased estimates.

5. Extended Kalman filtering – UD factorisation

The UD factorisation filtering algorithm is used as a basis to study the problem of parameter estimation for unstable/augmented systems. The advantage of this algorithm for parameter estimation is that it can handle process as well as measurement noise. Simultaneous

Table 8. Correlation matrix (example 2).

Constant	α	q	v/v_0	δe
1.0	−0.77	−0.25	0.99	−0.23
−0.77	1.0	0.58	−0.73	0.07
−0.25	0.58	1.0	−0.17	−0.74
0.99	−0.73	−0.17	1.0	−1.30
−0.23	0.07	−0.74	−0.30	1.0

Table 9. Variance proportions (example 2).

Constant number	Constant	α	q	v/v_0	δe
1	0	0.0005	0.0002	0.1152	0.0043
11.99	0	0.026	0.0	0.002	0.328
51.78	0	0.35	0.03	0.014	0.197
312.5	0	0.49	0.89	0.002	0.382
1826	1.0	0.14	0.07	0.87	0.088

estimation of states and parameters is achieved by augmenting the state vector with unknown parameters and applying the filtering algorithm to the augmented nonlinear model. The general nonlinear system is described by the following equations:

$$\dot{x}(t) = f(x, u, \Theta) + Gw(t);\ x(0) = x_0, \tag{48}$$

$$y(t) = h(x, u, \Theta), \tag{49}$$

$$z(k) = y(k) + v(k) \quad k = 1, 2, \ldots, N, \tag{50}$$

where x is the state vector, u the input vector, y the output vector, z the output measurement vector, w the process noise vector, v the measurement noise vector, Θ the unknown parameter vector, and w and v are the process and measurement noises, assumed to be Gaussian with zero mean and covariance matrices Q and R. The estimation algorithm is

Table 10. Parameter estimates (example 2).

		SNR				
		100		10		10
Parameter	True values	LS	LSME	LS	LSME	OEM*
Z_α/v_0	−0.771	−0.7378	−0.7401	−0.1580	−0.5406	−0.1084
$Z_{\delta e}$	−0.2989	−0.2588	−0.3000	−1.136	−0.3000	–
$Z_{v/v0}$	−0.1905	−0.6469	−0.1800	−0.1405	−0.1800	–
M_α	0.3794	0.5543	0.4340	−1.165	0.0159	0.7102
M_q	−0.832	−0.7014	−0.8052	−0.2064	−0.6472	−0.9886
$M_{\delta e}$	−9.695	−9.677	−9.599	−5.817	−9.596	−9.479
X_α	−0.9371	−0.2554	−0.2137	−0.2742	−0.2388	–
$X_{v/v0}$	−0.0296	0.1599	−0.0200	0.4790	−0.0200	–
$X_{\delta e}$	−0.0422	−0.0114	−0.0400	−0.0084	−0.0400	–
$M_{v/v0}$	0.0116	−1.904	0.012	1.946	0.012	–
$L1\%$	–	24.40	1.89	81.84	7.53	
$L2\%$	–	20.37	1.22	48.95	4.90	

*Convergence problems were encountered

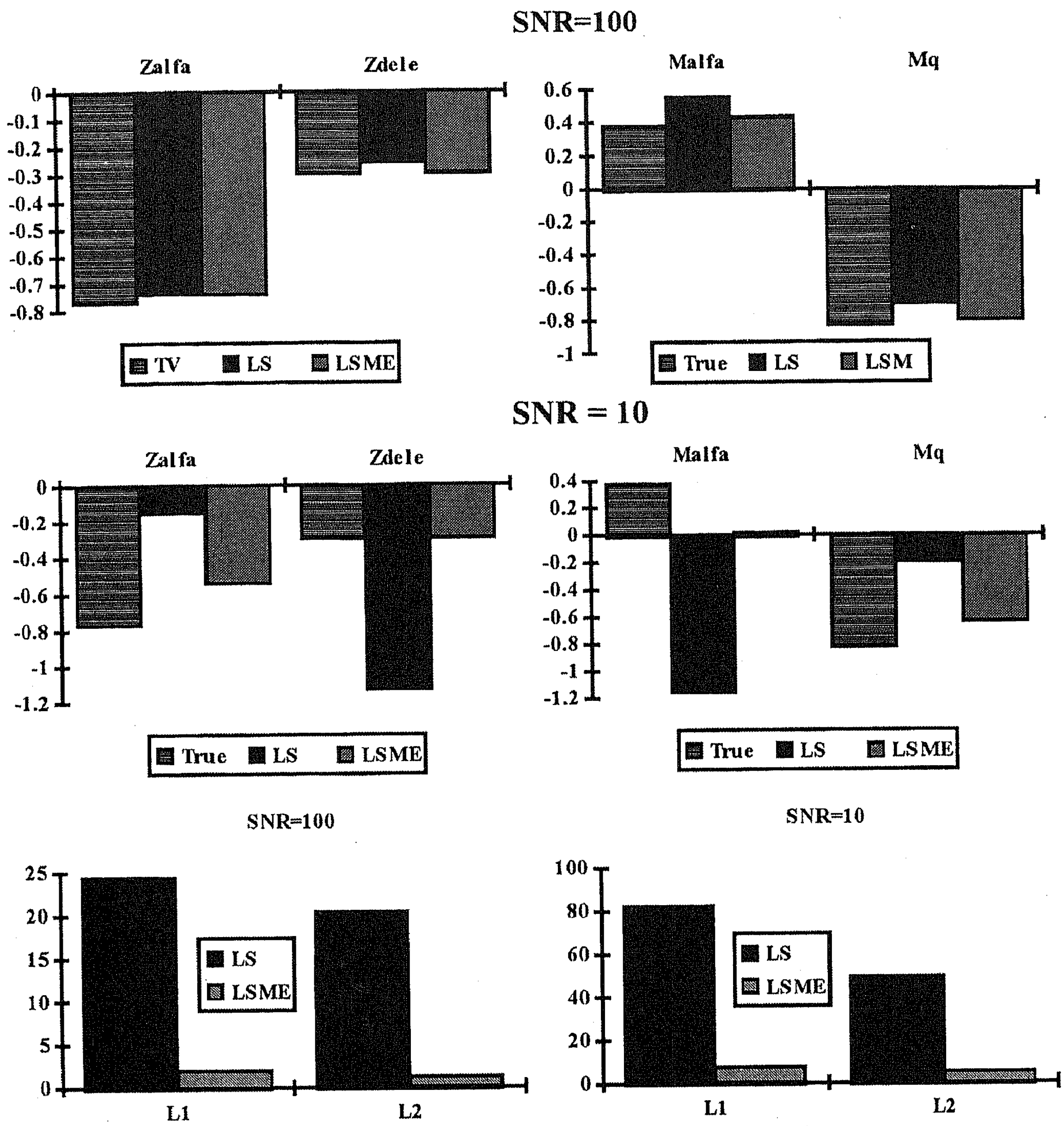


Figure 8. Comparison of parameters and PEENs (example 2, fourth-order longitudinal dynamics model).

obtained by linearising (49) and (50) around the prior/current best estimate of the state at each time and then applying the filtering algorithm to the linearised model.

5.1 *UD Kalman filter*

In the UD filter (Bierman 1977), the covariance update formulae and the estimation recursion are reformulated, so that the covariance matrix does not appear explicitly. Specifically, we use recursions for U and D factors of covariance matrix $P = UDU^T$ where U is unit upper triangular matrix and D is a diagonal matrix. Computing and updating with triangular matrices involve fewer arithmetic operations and thus greatly reduce the problem of roundoff errors, which might cause ill-conditioning and subsequent divergence of the algorithm. The filtering algorithm is given in two parts.

Time update: We have for the covariance update and the state prediction, the following equations,

$$\tilde{P}(k+1/k) = \Phi\hat{P}(K)\Phi^T + GQG^T, \tag{51}$$

$$\tilde{x}(k+1/k) = \Phi\hat{x}(k) + \psi u(k+1), \tag{52}$$

where Φ is the state transition matrix and Ψ is the discrete-time control matrix. Given $\hat{P} = \hat{U}\hat{D}\hat{U}^T$, the time update factors $\hat{U}$ and $\hat{D}$ are obtained through modified Gram–Schmidt orthogonalisation process (Biermann 1977).

Measurement update: The measurement update in Kalman filtering combines *a priori* estimate $\tilde{x}$ and error covariance $\hat{P}$ with a scalar observation $z = a'x + v$ to construct an updated estimate and covariance given as:

$$\hat{x} = \tilde{x} + K(z - a'\tilde{x}), \tag{53}$$

$$\hat{P} = \hat{P} - Ka\tilde{P}, \tag{54}$$

where $\alpha = a'\hat{P}a + \vartheta$; $K = \hat{P}a/\alpha$, $\tilde{P} = \tilde{U}\tilde{D}\tilde{U}'$, a is the measurement matrix, ϑ the measurement noise covariance and z the noisy measurements. The implementations of the filter are done in PC MATLAB.

5.2 *Identification of unstable systems*

In order to study the applicability of UD method to handle unstable data, a second-order system with varying degrees of instability is simulated. Table 11 gives the eigenvalues of the second-order plant for the various cases studied. For comparison purposes, the same set of data was analysed using OEM. The plant has the following state equations:

$$\dot{x}_{(2xl)} = A_{(2x2)}x_{(2x1)} + B_{(2x1)}u_{(1x1)}. \tag{55}$$

Figure 9 gives two of the 6 parameters of the A and B matrices, estimated using OEM and UD filter along with true values for the parameters for four cases listed in table 11. As the instability increases, despite very close start-up values, OEM does not converge to a stable solution and the iterations repeat indefinitely. Also, it was noted that as the instability increases, the standard deviations of the estimated parameters increase, the parameters are estimated with incorrect signs/magnitudes and correlation between the parameter estimates increases rendering inaccurate parameter estimates when OEM is used for analysis. With UD filter, the parameter convergence is good although with increasing

Table 11. Eigenvalues of the unstable second-order system.

Case No.	Eigenvalues
1	$0.0253 \pm (1.4140)$
2	$0.2250 \pm j(1.4043)$
3	$0.6250 \pm j(1.2920)$
4	$1.0250 \pm j(1.2956)$

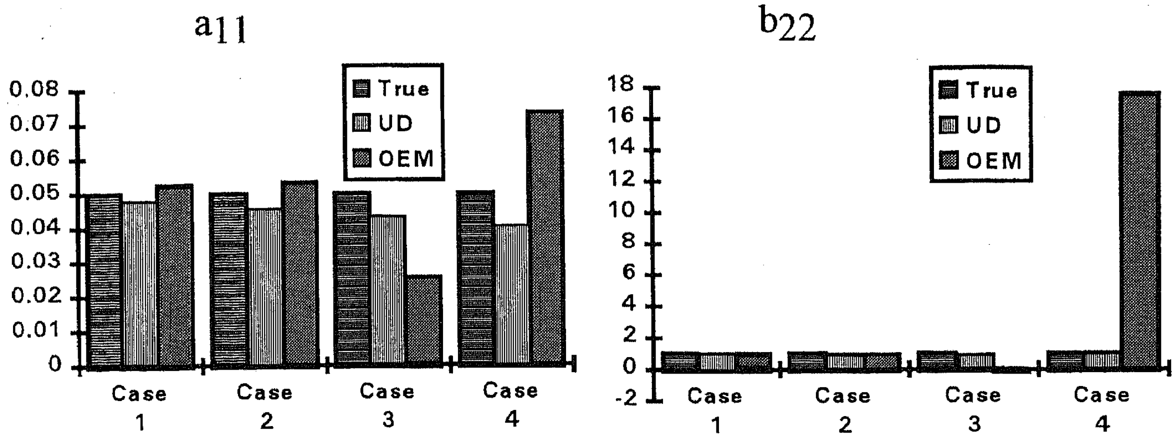


Figure 9. Parameter estimates (example 3, second-order system).

stability closer initial parameter estimates are required to ensure convergence. UD filter thus gives improved results for unstable systems because of the inherent stabilisation present in the filter. The feedback K from residual $(z - \hat{z})$ provides stability so that the residual error does not grow rapidly.

6. Closed loop identification

In many situations, the knowledge of the controllers used for stabilising the unstable aircraft is available. When the control system dynamics are known, they can be used to arrive at the total dynamic model of the closed loop system. Using the known values of the controller gain/dynamics in the model, the unknown parameters of the plant can be estimated. This method has been used for tunnel dynamic flying study in active control wherein the actual controller information is used in estimation of pitching moment derivatives of the standard dynamic model.

When the controller information is available, two approaches are possible to estimate the parameters of the open loop plant:

(i) Combined/equivalent parameters/model between the command (overall) input and the output of FBWCS-a/c are estimated. Since the feedback characteristics are known, the parameters of the a/c mathematical model are retrieved by simple algebraic transformation. This method is feasible when the controller is a simple one.

(ii) In general, FBWCS will be very complex and the retrieval of the a/c model parameters will not be feasible. For this case a combined/overall mathematical model of FBWCS-A/C is postulated. Only the aircraft mathematical model parameters are unknown and can be estimated. All the parameters of the FBWCS are kept fixed at their known values. This leads to a huge state-space model of the combined system. When complex controllers are used, the order of the system increases. In such cases, the model reduction methods could be used to arrive at a reduced order model. Because of the complexity involved, this approach is not pursued here.

Table 12. Parameter estimates-equivalent derivatives approach (example 3).

Parameter	True value	SNR $=\infty$	SNR $=10$
$Z_w + K_w Z_{\delta_e}$	−1.5945	−1.5819	−1.4968
Z_w	−1.4249	−1.4253*	−1.3402*
$M_w + K_w M_{\delta_e}$	−0.4196	−0.4199	−0.4057
M_w	−0.1	−0.1033*	−0.0861*
M_q	−3.7067	−3.7162	−3.3412
Z_{δ_e}	−6.2632	−6.3034	−5.9865
M_{δ_e}	−12.7840	−12.8004	−11.5989
$L1\%$	–	0.2875	7.9384
$L2\%$	–	0.3015	8.6171

* Retrieved from equivalent derivatives.

6.1 *Equivalent derivative estimation approach*

The approach of equivalent derivative estimation is described below for the case of the simple gain feedback around the second-order short period dynamics of an aircraft.

The simulated second-order system has the following state equations,

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_0 + Z_q \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e. \tag{56}$$

The w signal is fed back to the input through a gain K_w so that the control law is in this case defined as

$$\delta_e = K_w w + \delta_p, \tag{57}$$

where δ_p is the pilot command at J [1]. Inserting (57) into (56), we get the following augmented system state equations,

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w + K_w Z_{\delta_e} & Z_q \\ M_w + K_w M_{\delta_e} & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_p. \tag{58}$$

Thus we see that due to the augmentation, the coefficients in the first column of the matrix A are affected. The mathematical model for parameter estimation could be formulated as

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_{eq} & Z_q \\ M_{eq} & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_p, \tag{59}$$

and the parameters estimated. The derivatives Z_w and M_w of the plant can be computed from Z_{eq} and M_{eq} using the known value of the feedback gain K_w, Z_{δ_e}, M_{δ_e}. For this case input noise at 'K' (in figure 1) is not considered.

Example 3. Simulated data with $K_w = 0.025$ and SNR $=\infty$ and SNR $=10$ is used to estimate the equivalent model of (65). Table 12 gives the equivalent derivatives estimated using OEM. Since the feedback gain is known, the parameters of the plant are retrieved from the equivalent derivatives. From table 12 it is clear that parameter estimates are fairly accurate even in the presence of noise. This establishes that for known and simple feedback loops, the plant parameters can be retrieved with reasonable accuracy.

7. Concluding remarks

A review of various estimation methods for unstable/augmented aircraft is presented in this paper. Collinearity diagnostics are used to detect collinearity in simulated data and mixed estimation is used as a method to handle such data. LSME method for unstable/augmented systems and UD filter for unstable systems is validated. The details and numerical validation of the algorithm are given. Also, in situations where the control system details are known, they can be utilised in the aggregate model of the system leading to the controller information identification method (CIBIM). One of the variants of this is to estimate equivalent derivatives of the entire system and use the knowledge of the controller to retrieve the open loop dynamics. This method is shown to work well for simple feedback loops.

References

Belsley D A, Kuh E, Welsh R E 1980 *Regression diagnostics: Identifying influential data and sources of collinearity* (New York: John Wiley & Sons)

Bierman G J 1977 *Factorisation methods for discrete sequential estimation.* (London: Academic Press)

Girija G, Jategaonkar R V 1991 Some results of ATTAS flight data analysis using maximum likelihood parameter estimation method. DLR-FB-9104, IFM, Germany

Girija G, Raol J R 1993 PC based flight path reconstruction using UD factorisation filtering algorithm. *Defence Sci. J.* 43: 429–447

Girija G, Raol J R 1995a Controller information based identification methods. In *Proc. 34th Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, USA AIAA paper no. 96-0900

Girija G, Raol J R 1995b Estimation of parameters of unstable and augmented aircraft system using recursive mixed estimation technique. *J. Inst. Eng. (Aerosp. Div.)* 76: 15–22

Jategaonkar R V, Plaetchke E 1983 Maximum likelihood parameter estimation from flight test data. DFVLR-FB 83-14, IFM, Germany

Jategaonkar R V, Plaetschke E 1989 Algorithms for aircraft parameter estimation accounting for process and measurement noise. *J. Aircraft* 26: 360–372

Klein V 1989 Estimation of aircraft aerodynamic parameters from flight data. *Prog. Aerosp. Sci.* 26: 1–77

Koehler R, Wilhelm K 1979 Closed loop aspects of aircraft identification. AGARD LS-104, pp 10-1 to 10-25

Maine R E, Iliff K W 1986 Application of parameter estimation to aircraft stability and control – the output error approach. NASA RP 1168

Maine R E, Murray J E 1988 Application of parameter estimation to highly unstable aircraft. *J. Guidance, Control Dynamics* 11: 213–219

Mereau P, Abu El Ata-Doss S 1985 Parameter estimation of aircraft with fly-by-wire control systems. *IFAC Proceedings on identification and system parameter estimation* (Pergamon)

Plaetschke E, Weiss S, Rohlf D 1993 Identification at high angles of attack applied to X-31A flight test data. In *Proc. NAL DLR Symposium on System Identification*, DLR-Mitt. 93-14, DLR, Germany